

### Thin Elliptic Cone, $\tau = 0.40$

On the right-hand side of Fig. 2a, the flow visualization obtained on the thin elliptic cone again shows, in top view, the familiar vortex system of three vortices on each side of the cone. The primary vortices show a considerable degree of asymmetry; they are turned the opposite way (from a fictitious symmetry configuration) as was the case on the other two cones. Their degree of asymmetry has decreased as compared with the thicker cone of  $\tau = 0.65$ .

### Thin Delta Wing

On the left-hand side of Fig. 2a, the flow visualization obtained on the delta wing exhibits only one vortex, the primary vortex, on each side of the wing. Each of the well-structured vortices terminates in a bubble-like breakdown region, which is followed by irregular flow. The breakdown regions are located clearly asymmetrically with respect to the wing's symmetry plane. From the front of the wing the concentrated, well-structured vortices are nominally symmetric down to the first breakdown bubble. The nonexistence of asymmetric vortex flows on slender delta wings has been discussed in detail in Refs. 4 and 5.

### Elliptic Cone, $\tau = 1.54$

The elliptic cone with the thickness ratio  $\tau = 0.65$  was turned through an angle of 90 deg to provide an elliptic cone with  $\tau = 1.54$ . The flow visualizations obtained are not presented here, only discussed. In the initial tests performed it was observed that the vortex system on the leeside was strongly oscillating in the spanwise direction. Two extreme configurations were observed: The flow was first swept to the left side (looking downstream) over the rear part of the cone, while at some later state, it has become largely symmetric. From the side, it was observed that one of the vortices intermittently lifted off in the rear part, while the other vortex followed the cone all of the way down to the base. It was found that one extreme configuration of the vortex system is a large liftoff from the cone of the vortex, combined with an extreme spanwise displacement of the vortex system over the rear portion of the cone. The other extreme configuration exists with the main vortices along the cone and in a practically symmetric arrangement.

In a second part of the tests, then, extra attention was given to establish a low disturbance level flow in the empty test section. To this end, the water was circulated in the tunnel (without the model installed) for a relatively long time (approximately 1.5 h). Then the model was brought into the test section with utmost care to minimize disturbance of the flow. Between two tests, periods of water circulation, without the model, of about 5 min were allowed to reduce the disturbance level.

Flow observations from the side now revealed only occasional, slight vortex liftoff over the rear part of the cone; otherwise the primary vortices lie along the cone's leeside surface down to the base. In the top view, the vortex system was observed to distinctly oscillate from one side to the other over the aft portion of the cone, being strongly asymmetric in most cases; no temporarily steady, symmetric configuration was observed. In this test series, with apparently reduced freestream disturbance levels, the (practically) maximum liftoff of a primary vortex is much reduced as compared to that observed under normal operating conditions of the water tunnel. This must be taken into account in further tests.

## V. Conclusions

Flow-visualization tests were carried out on three elliptic cones and a delta wing, with thickness ratios  $\tau$  varying between  $\tau = 1$  (circular cone) and  $\tau \rightarrow 0$  (delta wing) and one inverted cone  $\tau = 1.54$ , at one angle of incidence  $\alpha = 38$  deg, and Reynolds number  $Re_L = 2.8 \times 10^4$ . The main result of the study is that the degree of asymmetry of the vortex flows behind the cones decreased as the cones became flatter, i.e., with  $\tau$  decreasing from  $\tau = 1$  to 0.4, and was zero for the delta wing. Our limited experimental results are in qualitative agreement with the results of an inviscid-flow theory by Fiddes and Williams. The outcome of the study suggests an attempt to modify a conventional, thick, axisymmetric missile nose toward a flatter nose, e.g., with elliptic cross sections, to reduce flow asymmetry at high incidence. Such a nose geometry has been proposed in Ref. 11 and will be studied at realistic Reynolds numbers. Furthermore, it has been observed in a specific test

series, with apparently reduced freestream disturbance levels, that the liftoff of a vortex from a cone is distinctly reduced, as compared to that under normal tunnel operating conditions. This has to be taken into account in future testing

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## Relation Between Spectra of Hot-Wire Signals and Velocity Fluctuations

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## Introduction

ONE of the most common methods of measuring turbulence is by the use of a hot-wire anemometer. In a hot-

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wire anemometer, there exists a nonlinear relationship between the instantaneous velocity in the flow and the instantaneous voltage output of the instrument. Just as the velocity is comprised of a constant mean and fluctuating component, the voltage output of the instrument can also be decomposed into mean and fluctuating components. It is common practice, particularly in earlier times when analog data processing was prevalent, to obtain the spectrum of the fluctuating hot-wire anemometer voltage output and consider it as representative of the spectrum of the fluctuating velocity. In view of the nonlinear relationship between velocity and hot-wire anemometer output voltage, there are some scale factors that need to be used while relating the two spectra. No discussion is found in the literature regarding converting the spectrum of the voltage signal into that of the velocity. Moreover, if a cross-wire probe is used, the analysis becomes more complex since the spectrum of the cross correlation would also be involved. In this paper, therefore, a complete analysis of the relation between spectra of hot-wire anemometer fluctuating voltages and those of the turbulence quantities is presented for both single-wire and cross-wire probes. Experimental results are presented to support the analysis.

Spectral analysis of turbulence data can be performed by applying either classical techniques that use the discrete Fourier transform<sup>1</sup> or a set of new techniques known as parametric ones.<sup>2</sup> Among the several techniques currently available for digital spectral analysis, the question naturally arises as to which spectral analysis technique works well with an actual flow process represented by the sampled digital data. A comparative study was, therefore, made by Rajan and Munukutla<sup>3,4</sup> to assess the performance of three different techniques for the estimation of the turbulence energy spectrum. In this paper, the correlogram technique was used for the spectral analysis.

### Analysis for a Single-Wire Probe

The instantaneous velocity of the flow  $U_i$  and instantaneous voltage output  $E_i$  of the hot wire can be decomposed into their mean and fluctuating components as  $\bar{U} + u_i$  and  $\bar{E} + e_i$ , respectively, and their relation is

$$(\bar{E} + e_i)^2 = A + B\sqrt{\bar{U} + u_i} \quad (1)$$

where  $A$  and  $B$  are the calibration constants.

For low-turbulence intensities,

$$(\bar{E})^2 = A + B\sqrt{\bar{U}} \quad (2)$$

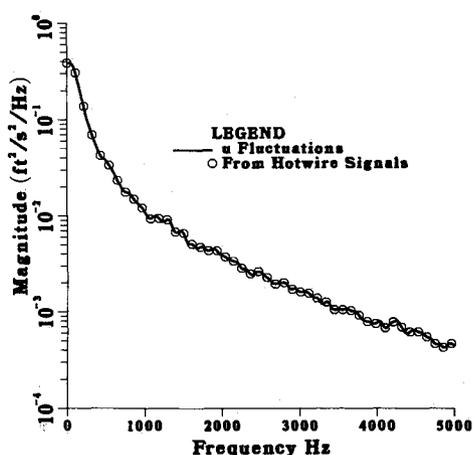


Fig. 1 Comparison of spectra of  $u$  from hot-wire signals with scale factor and from  $u$  fluctuations for a single-wire probe.

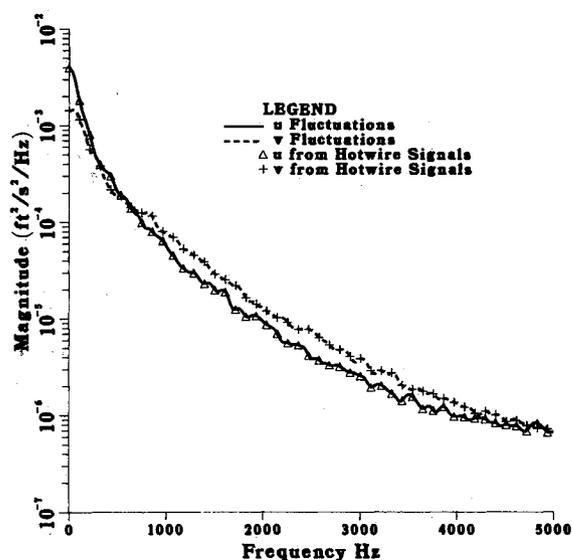


Fig. 2 Comparison of spectra of  $u$  and  $v$  from hot-wire signals with scale factors and from velocity fluctuations for a cross-wire probe.

and

$$u_i = \frac{e_i}{s} \quad (3)$$

where

$$s = \frac{B}{4E\sqrt{U}} \quad (4)$$

Note that Eqs. (1–4) are linearized equations for a single-wire probe and are available from standard textbooks, for example, Hinze.<sup>5</sup>

Considering Eq. (3), we can relate the spectra of  $u_i$  and  $e_i$  by taking the Fourier transform of the autocorrelation. Thus,

$$S_u(f) = \frac{1}{s^2} S_e(f) \quad (5)$$

where  $S_u(f)$  is the spectrum of  $u_i$  and  $S_e(f)$  the spectrum of  $e_i$ . The result given in Eq. (5) is verified by applying it to turbulence data in the central region of a circular duct where the turbulence level is about 3%. Experiments were conducted in a 6-in.-diam tube at a Reynolds number of  $2 \times 10^5$ . Due to this low turbulence level, the linearization assumptions made in deriving Eq. (3) are justified. Depicted in Fig. 1 is the spectrum of the  $u$  fluctuations shown as a solid line with the corresponding spectrum of the voltage fluctuation divided by  $s^2$  as discrete points. The spectrum of the fluctuating voltage  $e_i$  was obtained by directly sampling the hot-wire signal at 20 kHz and performing spectral analysis using the correlogram method. In order to obtain the spectrum of  $u_i$ , the sequence of  $u_i$  had to be generated in the following manner. Each instantaneous  $e_i$  were added to  $\bar{E}$  and then converted to  $U_i$  by using Eq. (1). The  $U_i$  were then averaged to find  $\bar{U}$  and each  $u_i$  obtained by subtracting  $\bar{U}$  from each  $U_i$ . Thus, the sequence of  $u_i$  corresponding to the sequence of  $e_i$  was generated and spectral analysis performed on the sequence of  $u_i$  by using the same correlogram technique.

### Analysis for a Cross-Wire Probe

In the case of a cross-wire probe, the voltage output from each wire is influenced by both  $u_i$  and  $v_i$ . The velocity-voltage relationships can be written as

$$(\bar{E}_1 + e_{1i})^2 = A_1 + B_1\sqrt{\bar{U} + u_i + v_i} \quad (6)$$

$$(\bar{E}_2 + e_{2i})^2 = A_2 + B_2\sqrt{\bar{U} + u_i - v_i} \quad (7)$$

where  $A_1$ ,  $B_1$  and  $A_2$ ,  $B_2$  are the calibration constants for wires 1 and 2, respectively. By performing a linear analysis similar to that of a single wire, we obtain expressions for  $u_i$  and  $v_i$  as

$$u_i = ae_{1i} + be_{2i} \quad (8)$$

$$v_i = ae_{1i} - be_{2i} \quad (9)$$

where

$$a = \frac{2\bar{E}_1\sqrt{\bar{U}}}{B_1} \quad (10)$$

and

$$b = \frac{2\bar{E}_2\sqrt{\bar{U}}}{B_2} \quad (11)$$

Equations (8) and (9) indicate that  $u_i$  and  $v_i$  are functions of  $e_{1i}$  and  $e_{2i}$ . The spectra of  $u_i$  and  $v_i$  are obtained by taking the Fourier transform of the autocorrelation. From Eqs. (8) and (9), we can write

$$S_u(f) = a^2 S_{e_1}(f) + b^2 S_{e_2}(f) + ab [S_{e_1 e_2}(f) + S_{e_2 e_1}(f)] \quad (12)$$

$$S_v(f) = a^2 S_{e_1}(f) + b^2 S_{e_2}(f) - ab [S_{e_1 e_2}(f) + S_{e_2 e_1}(f)] \quad (13)$$

where  $S_{e_1}(f)$  and  $S_{e_2}(f)$  are the spectra of  $e_1$  and  $e_2$ , respectively, and  $S_{e_1 e_2}(f)$  and  $S_{e_2 e_1}(f)$  are the cross-spectra of  $e_1$  and  $e_2$ .

The foregoing relations were verified by analyzing turbulence data in the central region of a circular duct obtained with

a cross-wire probe. The spectra of the fluctuating voltages were obtained by employing the correlogram technique on the digitally sampled signals. Also from the voltages, the velocities were computed using relations (6) and (7) and the spectra of the velocities using the correlogram method. The results are shown in Fig. 2. The  $e_1 e_2$  and  $e_2 e_1$  spectra have a significant influence on the spectra of  $u$  and  $v$ .

### Conclusion

The relation between spectra of hot-wire fluctuating voltages and the spectra of velocity fluctuations for both single- and two-wire probe were derived. The results were verified by applying them to experimental data for flow in a circular duct. It was found that in the case of a two-wire probe, the cross-spectra of  $e_1$  and  $e_2$  play an important role.

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## Errata

### Linearized Euler Predictions of Unsteady Aerodynamic Loads in Cascades

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**D**URING typesetting of this paper, several errors were inadvertently introduced. We regret these errors.

#### Page 545

Column 2, line 17 should read, "These modes are spurious computational modes with no physical counterparts."

#### Page 548

Column 2, line 15 should refer to Ref. 27 rather than Ref. 26.

#### Page 550

References 22 through 27 should be renumbered as follows:

- <sup>22</sup>Hall, K. C., and Lorence, C. B., "Calculation of Three-Dimensional Unsteady Flows in Turbomachinery Using the Linearized Harmonic Euler Equations," American Society of Mechanical Engineers, 37th International Gas Turbine and Aeroengine Congress and Exposition, Paper 92-GT-136, Cologne, Germany, June 1-4, 1992.
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